

# Fluctuations as a test of chemical non-equilibrium at the LHC

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It is shown that large chemical potential leads to the significant increase of multiplicity fluctuations for bosons, and makes the fluctuations infinite in the case of Bose-Einstein condensation. It allows to distinguish between the models that explain the anomalous proton to pion ratio and the low transverse momentum enhancement of pion spectra in Pb+Pb collisions at the LHC within chemical equilibrium or non-equilibrium models. The effects of resonance decays, finite size of the system, requirements to the event statistics, different momentum cuts, and limited detector acceptance are considered. The obtained results show the possibility to observe a substantial increase of the normalized kurtosis for positively or negatively charged pions in the case of non-equilibrium or partial pion condensation using currently measured data.

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## I. INTRODUCTION

During last decades thermal model [1–10] (TM) became a standard tool for the analysis of mean multiplicities in nucleus-nucleus collisions. It is implemented in free online codes [11–13], and obtained temperatures are discussed as a basic property of a created system in the papers reporting experimental results, see e.g. [14, 15]. The temperatures follow a smooth freeze-out line in a wide energy range of colliding nuclei [16, 17]. The initial energy of the collision is 10 times larger at the LHC (Large Hadron Collider) than at RHIC (Relativistic Heavy Ion Collider) and 100 times larger than at SPS (Super Proton Synchrotron). The temperature grows with increasing energy of the collision, and was expected to saturate around  $T \simeq 165$  MeV. Therefore, the LHC data [14, 18] came as a big surprise, because their description requires that the temperature falls down from the freeze-out line by 10 MeV [15, 19], being smaller than at RHIC and close to that at the SPS<sup>1</sup>. This difference in temperature is very large for a TM, because all particles except for pions have the mass  $m \gg 165$  MeV, and their mean multiplicities in TM depend exponentially on temperature  $\langle N \rangle \sim \exp[-m/T]$ .

Besides the lower temperature at the LHC, there are substantial difficulties in simultaneous description of pions, protons and strange particles. Proton to pion ratios are suppressed at the LHC compared to RHIC [14]. Experimentally measured pion spectra at the LHC rise steeper for low transverse momentum  $p_T$  than in the models [22–24], while the same models work perfectly at RHIC. There are many ways to explain the proton to pion ratio or strange particles [25–30], but the low  $p_T$  enhancement of the pion spectrum at the LHC, is still the open problem.

Both, proton to pion ratio and the low  $p_T$  pion spectrum can be explained in the non-equilibrium TM [31, 32]. It allows for a non-equilibrium chemical potential<sup>2</sup> for each particle, due to partial equilibration of the constituent quarks in the fast expanding fireball [33, 34]. This model has two more parameters compared to the standard TM - one for light and one for strange quarks. The numerical calculations in the non-equilibrium TM give even smaller temperature  $T \simeq 140$  MeV, and the large positive chemical potential for pions close to its mass  $\mu_\pi \simeq m_\pi$  [31], see also [35]. This may imply the Bose-Einstein condensation (BEC) of pions [36–38].

The pion chemical potential was introduced to explain the early data at the SPS [39], and was similarly justified by partial thermalization [40], and also by pion condensation [41–43]. However, the update of the resonance list gave the same effect [5], and pion BEC was abandoned. It seems not to be the case at the LHC, because the properties of the resonances with  $m < 2.5$  GeV are known very well now. They are already included in TM, and do not give the required amount of low  $p_T$  pions. The resonances with  $m > 2.5$  GeV may give the effect, if the particular type of Hagedorn-like states that decay mainly into low  $p_T$  pions exist. The deficiency of pions at the LHC is observed at the  $p_T \leq 150$  MeV [37]. It means that the not yet observed Hagedorn-like

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<sup>1</sup> The recent analysis of the new SPS data gives a different freeze-out line, which points to the low LHC temperature [20]. The same TM used for the LHC confirms this finding [21].

<sup>2</sup> The non-equilibrium can describe proton spectra as good as the rescattering [23], but only a non-equilibrium chemical potential can give the low  $p_T$  enhancement of pions seen in the data [32].

states with  $m > 2.5$  GeV should decay through multi-pion channels with 5-10 pions, or through a particular sequential decay, that gives many low  $p_T$  pions. The only possible light meson candidate was the famous sigma  $f_0(500)$  meson, but it should *not* be included in TM at all [44–47].

There are good reasons for chemical non-equilibrium with  $\mu_\pi > 0$  at the LHC. It was predicted in the super(over) cooling scenario [48, 49]. The extra pions at low  $p_T$  may appear due to fast hadronization of the gluon condensate [50, 51], glueballs [52], or Color Glass Condensate [53, 54], forming transient Bose-Einstein condensate of pions [55]. The time needed to form such a condensate at the LHC is lower than at RHIC, and is just  $t \sim 0.1 - 0.2$  fm/c [56, 57]. The analysis of two-, three- and four-particle correlations by ALICE Collaboration [58, 59] gives large values for the amount of pions from a coherent source - 20 – 30%. They do not specify the nature of the coherent emission, but pion condensate is a good candidate.

A large positive chemical potential substantially increases multiplicity fluctuations of bosons and makes the fluctuations *infinite* for the case of pion BEC in an infinite system [60, 61]. It may allow to differ pion BEC from other effects like the production of Hagedorn-like states discussed above, or the disoriented (disordered) chiral condensate (DCC), see e.g. [62, 63]. At the LHC the radius of the system at freeze-out is  $r \sim 10$  fm [64], and the amount of pions on the zero momentum (condensate) level might be around 5% [37]. However, it can be enough to observe a detectable signal in pion multiplicity fluctuations<sup>3</sup>, using currently measured events by ALICE. If the fluctuations will be found small, then it will be a strong argument against non-equilibrium at the LHC. However, if the fluctuations will be found large, then one could use them as a tool to study the non-equilibrium.

The paper is organized as follows. In Section II the phase diagram of the pion gas is obtained in order to determine the centrality where the largest amount of the condensate is possible at the LHC. In Section III the fluctuations of primary pions are calculated, and further suggestions how to search for the condensate are formulated. In Section IV the resonance decay contribution, requirements to event statistics, and the effects of limited detector acceptance are considered. A specific  $p_T$  cut is proposed to enhance the effect of possible pion condensation. Section V concludes the paper.

## II. PHASE DIAGRAM OF THE CONDENSATE

Bose-Einstein condensation is possible at any temperature, if the density of bosons  $\rho$  is high enough

$$\rho(T, \mu) = \frac{V}{(2\pi)^3} \int \frac{d^3p}{\exp[(E_p - \mu)/T] - 1}, \quad (1)$$

where  $V$  is the system volume, and  $E_p = \sqrt{p^2 + m^2}$  is the energy of a boson with a momentum  $p$ . The critical density is defined in TM as  $\rho_C(T) = \rho(T, \mu = m)$ , that gives a continuous condensation line in the  $T - \rho$  plane [61, 74]. Therefore, one can also find the condensation temperature  $T_C$  for each density  $T_C(\rho)$ .

Multiplicity fluctuations rise to infinity at the condensation line in the infinite volume limit [61], and increase fast in it's vicinity for a finite volume of the system [74]. Therefore it is important to know how far the system is from the condensation line. The finite volume corrections were implemented in [36] to SHARE model [13]. The corresponding fit of the mid-rapidity yields  $\frac{dN_i}{dy}|_{|y|<0.5}$  at the LHC confirms that chemical potential is relevant only for pions, giving a smaller value than in [31]. In this model chemical potential is the same for charged and neutral pions, so neutral pions could 'feel' the condensation effects at smaller  $\mu_\pi$ , due to lower mass. However, their multiplicity is not measured yet, and the spectrum is available only for  $p_T > 700$  MeV [75], while any effect of the condensate on spectra can be seen for much smaller momenta  $p_T < 200$  MeV [37]. Moreover, in order to address fluctuations, the number of particles should be measured event-by-event, which is even more complicated. The number of positively and negatively charged pions is the same within the error bars [14], therefore  $\mu_{\pi^+} \simeq \mu_{\pi^-} \equiv \mu$  and there is no difference which one to use. However, charge identification is important, because  $\pi^+$  and  $\pi^-$  are different particles that condense separately.

The densities and chemical potentials for positively or negatively charged pions are calculated for different centralities of the collision at the LHC, using the parameters obtained in [36], and are shown in Fig. 1. One can see that  $\rho < \rho_C$  and  $T > T_C$  at the LHC, so the condensate line is not reached, but central and semi-central collisions with centrality  $c < 40\%$  are the closest to the condensation line. Note a small temperature for the most central collisions  $T \simeq 140$  MeV as in [31, 76], which increases for peripheral collisions and reaches the

<sup>3</sup> The high order fluctuations received a lot of attention recently due to a possibility to detect QCD critical point, see e.g. [65–73], however, it seems that pion fluctuations with  $\mu_\pi \gg 0$  were not studied yet.

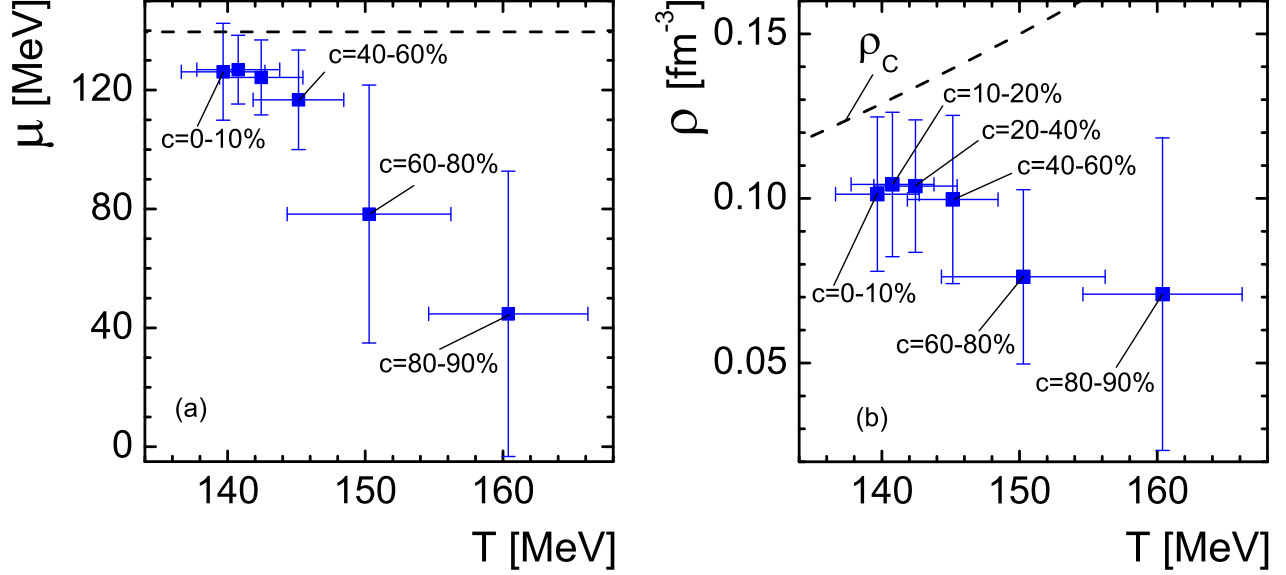


FIG. 1: (a): The pion chemical potential and temperature at freeze-out for different centralities in the 2.76 TeV Pb+Pb collisions at the LHC in the non-equilibrium TM [36]. (b): The same for densities and temperatures. The dashed lines show the chemical potential  $\mu = m_{\pi^\pm}$  and the critical density  $\rho_c(T)$ .

equilibrium TM result of  $T \simeq 150 - 160$  MeV [19, 20] for very peripheral collisions. The chemical potential decreases for peripheral collisions in contrast to [31], because of the finite size of the system at freeze-out. Therefore, the condensation is more probable in the most central collisions, where the system is also spatially larger and lives longer.

The error bars are obtained using the standard methods for propagation of uncertainty. The necessary correlations of the parameters are calculated for the 10% deviation of the  $\chi^2/N_{\text{dof}}$  from the best fit [37]. The correlation between all pairs of thermodynamic parameters is negative at all centralities, except for a small positive correlation between  $V$  and  $T$  at  $c = 60 - 80\%$ , and between  $T$  and  $\mu_\pi$  at  $c = 80 - 90\%$ . Therefore, the error bars are the largest at these centralities. However, they are significant also at other centralities. It reflects the freedom in choosing the parameters to fit the available data. A larger set of measured mean multiplicities should decrease this ambiguity.

### III. FLUCTUATIONS OF PRIMARY PIONS

Multiplicity fluctuations of any order can be calculated for primary pions analytically, using the definition of susceptibilities  $\chi_n$ . They are given by the derivatives of pressure  $\mathcal{P}$  by chemical potential  $\mu$  at constant temperature  $T$ , see e.g. [71, 77]

$$\chi_n = \frac{\partial^n (\mathcal{P}/T^4)}{\partial (\mu/T)^n} \Big|_T. \quad (2)$$

The pressure in the pion gas is given by

$$\mathcal{P}/T^4 = \frac{1}{T^3} \sum_p \ln(1 - \exp[(\mu - E_p)/T])^{-1}, \quad (3)$$

The convenient measures are the scaled variance (variance over the mean)  $\omega = \sigma^2/\langle N \rangle$ , normalized skewness  $S \cdot \sigma$ , and normalized kurtosis<sup>4</sup>  $\kappa \cdot \sigma^2$ . They are directly related to the susceptibilities and central moments

$$\omega = \frac{\chi_2}{\chi_1} = \frac{m_2}{\langle N \rangle}, \quad S \cdot \sigma = \frac{\chi_3}{\chi_2} = \frac{m_3}{m_2}, \quad \kappa \cdot \sigma^2 = \frac{\chi_4}{\chi_2} = \frac{m_4}{m_2} - 3m_2, \quad (4)$$

<sup>4</sup> Note that for Gauss (Normal) distribution  $\omega$  can get any value, while  $S \cdot \sigma = \kappa \cdot \sigma^2 \equiv 0$ .

where  $\langle N \rangle$  is the mean multiplicity and

$$m_n = \langle (N - \langle N \rangle)^n \rangle = \sum_N (N - \langle N \rangle)^n \cdot P(N) \quad (5)$$

are the central moments of the  $P(N)$  multiplicity distribution. Equation (2) is very useful for theoretical calculations, while Eq. (5) is better for experimentalists, because they directly measure the  $P(N)$ . The straightforward calculation using Eqs. (2)-(4) give:

$$\langle N \rangle = \sum_p \langle n_p \rangle, \quad (6)$$

$$\omega = \frac{\sum_p (\langle n_p \rangle + \langle n_p \rangle^2)}{\sum_p \langle n_p \rangle} = 1 + \frac{\sum_p \langle n_p \rangle^2}{\sum_p \langle n_p \rangle}, \quad (7)$$

$$S \cdot \sigma = \frac{\sum_p (\langle n_p \rangle + 3\langle n_p \rangle^2 + 2\langle n_p \rangle^3)}{\sum_p (\langle n_p \rangle + \langle n_p \rangle^2)}, \quad (8)$$

$$\kappa \cdot \sigma^2 = \frac{\sum_p (\langle n_p \rangle + 7\langle n_p \rangle^2 + 12\langle n_p \rangle^3 + 6\langle n_p \rangle^4)}{\sum_p (\langle n_p \rangle + \langle n_p \rangle^2)}, \quad (9)$$

where  $\langle n_p \rangle^k = \left\{ \exp \left[ (\sqrt{p^2 + m^2} - \mu)/T \right] - 1 \right\}^{-k}$ .

In equilibrium  $\mu = 0$ , and Eqs. (7)-(9) give for positively (negatively) charged pions at  $T = 140$  MeV:

$$\omega \simeq 1.1, \quad S \cdot \sigma \simeq 1.2, \quad \kappa \cdot \sigma^2 \simeq 1.9. \quad (10)$$

For  $\mu = 0$  and  $m/T \rightarrow \infty$  one recovers the result for Boltzmann statistics with  $\omega = S \cdot \sigma = \kappa \cdot \sigma^2 = 1$ . The  $\mu = 0$  and  $m/T \rightarrow 0$  is never realized in TM, because the temperatures are usually of the order of the pion mass or lower. However, one can see that for this case the scaled variance is finite,  $\omega \simeq 1.368$  [60], but  $S \cdot \sigma$  and  $\kappa \cdot \sigma^2$  diverge on the lower bound of the momentum integral, that usually replaces the sum over the momentum levels in (6)-(9)  $\sum_p \rightarrow V/(2\pi^2) \int p^2 dp$ . For  $\mu \rightarrow m$  even  $\omega$  diverges, as well as all  $\int p^2 dp \langle n_p \rangle^k$  with  $k \geq 2$ . This is the consequence of the fact that Bose-Einstein condensation is the 3rd order phase transition<sup>5</sup>. However, there are no divergences in finite volume, because the maximal fluctuations are bounded by the number of particles in the system. One can take the finite volume into account keeping the zero momentum state in the sum  $\sum_p$

$$\sum_p \langle n_p \rangle^k \rightarrow \langle n_0 \rangle^k + \frac{V}{2\pi^2} \int_0^\infty \langle n_p \rangle^k p^2 dp, \quad (11)$$

because  $\langle n_0 \rangle = \frac{1}{\exp[(m-\mu)/T]-1}$  grows as fast as volume in the limit  $\mu \rightarrow m$  [74]. The corresponding competition between  $\mu$  and  $V$  during the fit of pion mean multiplicities led to the decrease of pion chemical potential in [36, 37] compared to [31]. The relative contribution of the first term on the right hand side of Eq. (11) is larger for  $\mu \rightarrow m$ , because the largest contribution to the integral comes from the lower bound  $p \rightarrow 0$ , which diverges as  $\langle n_0 \rangle^k$  in this limit, but the  $p^2 dp \rightarrow 0$  weakens the divergency. Therefore, at  $\mu \rightarrow m$  one can estimate the fluctuations assuming that there is only the condensate level  $p = 0$ . Keeping also only the highest  $k$  in Eqs. (7)-(9) gives

$$\omega \simeq \frac{1}{\langle N \rangle} \frac{1}{\delta^2}, \quad S \cdot \sigma \simeq \frac{2}{\delta}, \quad \kappa \cdot \sigma^2 \simeq \frac{6}{\delta^2} \quad (12)$$

where  $\delta = (m - \mu)/T$ . The approximation (12) is valid, if  $\langle n_0 \rangle^2 \gg \langle N \rangle$ , i.e. for  $\omega \gg 1$ . Let us also assume for simplicity that the condensation line is already reached, because it excludes the  $\rho - \rho_C$  dependance. Then,  $\delta = (aV)^{-2/3}$ , where  $a = (mT)^{3/2}/(\sqrt{2}\pi)$  [74] and one obtains

$$\omega \simeq \frac{a}{\rho_C} (aV)^{1/3}, \quad S \cdot \sigma \simeq 2(aV)^{2/3} \sim \omega^2, \quad \kappa \cdot \sigma^2 \simeq 6(aV)^{4/3} \sim \omega^4. \quad (13)$$

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<sup>5</sup> The similar divergences in high order fluctuations measures take place close to critical point [78].

Therefore, the higher is the order of fluctuations, the faster they grow.

Equations (11,12) suggest that the fluctuations at  $\mu \rightarrow m$  should increase, if one finds a way to increase the relative amount of registered particles on the  $p = 0$  level, see [79]. It can be done by applying the  $p_T$  cut that selects more pions from the condensate  $\langle n_0 \rangle$ . The pion spectra at the LHC are measured starting from  $p_T > 100$  MeV. Pions on the  $p = 0$  level can receive a momentum  $p_T \lesssim 200$  MeV, because of the collective motion with the hypersurface [37]. Therefore, three distinct cases can be considered:

- all  $p_T$  - the easiest to calculate, but hard to measure,
- $p_T > 100$  MeV - currently measured data,
- $p_T = 100 - 200$  MeV - contains the highest percentage of pions from the  $p = 0$  level.

Fluctuations of primary pions, both normal and those from the condensate, can be calculated in Cracow single freeze-out model [80–83]. It can be done taking numerically the integral over the hypersurface, and for the corresponding  $p_T$  intervals  $\Delta p_T^{\text{norm}}$ , similar to the case with just the spectra in Ref. [37]

$$\begin{aligned} \sum_p \langle n_p \rangle^k &= \frac{1}{\{\exp[(m - \mu)/T] - 1\}^k} \frac{\Delta p_T^{\text{cond}}}{p_T^{\text{max}}} \\ &+ \frac{1}{(2\pi)^3} \int_{\Delta p_T^{\text{norm}}} p_T dp_T \int_0^{2\pi} d\phi_p \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta_{||} \int_0^{r_{\text{max}}} r dr \\ &\times \left[ m_T \sqrt{\tau_f^2 + r^2} \cosh(\eta_{||} - y) - p_T r \cos(\phi - \phi_p) \right] \\ &\times \left\{ \exp \left( \frac{1}{T} \left[ m_T \sqrt{1 + \frac{r^2}{\tau_f^2}} \cosh(\eta_{||} - y) - p_T \frac{r}{\tau_f} \cos(\phi - \phi_p) \right] - \frac{\mu}{T} \right) - 1 \right\}^{-k}, \end{aligned} \quad (14)$$

where the first term is the contribution from the condensate, and  $\Delta p_T^{\text{cond}}$  is the interval where the corresponding  $p_T$  cut overlaps with the condensate. The maximal momentum of the condensate,  $p_T^{\text{max}} = m r_{\text{max}}/\tau_f$ , is determined by the radius of the hypersurface  $r_{\text{max}}$  and the freeze-out time  $\tau_f$  [37];  $m_T = \sqrt{m^2 + p_T^2}$  is the transverse mass, and  $\int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta_{||} \int_0^{r_{\text{max}}} r dr$  is the integration over the hypersurface. The integral over all  $p_T$  gives the same as the integral over the volume per unit rapidity  $V = \pi r_{\text{max}}^2 \tau_f$  [64]. The integral over rapidity  $dy$  is absent in the right hand side of Eq. (14), because the fit of thermodynamic parameters was done for the rapidity densities  $\frac{dN_s}{dy}|_{|y|<0.5}$  [36, 37].

Another possibility to enhance the fluctuations is the increase of volume<sup>6</sup>, as seen from Eq. (13). It can be done by increasing the rapidity interval where pions are measured. It should be noted that the same assumption as in [37] is made so far, that the coherence length of the condensate in rapidity,  $\Delta y_{\text{cond}}$ , is the same as the rapidity interval of the measurements. If  $\Delta y_{\text{cond}}$  is much larger, then one could use the approximate formula (19) from the next section. If  $\Delta y_{\text{cond}}$  is smaller and fluctuations of the condensate come from the uncorrelated parts of the freeze-out hypersurface, or just from a small part of it, then the fluctuations will be smaller and scale differently from (13). In any case, the  $\kappa \cdot \sigma^2$  observable seems to be sensitive enough to study these effects.

#### IV. RESONANCE DECAY CONTRIBUTION

The question about fluctuations in a real system can be addressed semi-analytically under the assumption that the system consists of two parts that do not correlate. It seems to be a reasonable approximation, because the corresponding fluctuations are very different. As we will see, the fluctuations of pions from resonance decays in small acceptance window in rapidity are  $\sim 1$ , as for Poisson distribution. At the same time, pion fluctuations rapidly increase at  $\mu \rightarrow m$ , see Eqs. (12)-(13).

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<sup>6</sup> The rapidity distributions are flat in the wide range of rapidities at the LHC. Thus, temperature and chemical potentials should not change, while total volume increases with increasing the rapidity interval.

For two uncorrelated multiplicity distributions  $P_1(N_1)$  and  $P_2(N_2)$  one has:

$$\langle N \rangle = \langle N_1 \rangle + \langle N_2 \rangle , \quad (15)$$

$$\omega = \omega_1 \frac{\langle N_1 \rangle}{\langle N \rangle} + \omega_2 \frac{\langle N_2 \rangle}{\langle N \rangle} , \quad (16)$$

$$S \cdot \sigma = S_1 \cdot \sigma_1 \frac{\omega_1}{\omega} \frac{\langle N_1 \rangle}{\langle N \rangle} + S_2 \cdot \sigma_2 \frac{\omega_2}{\omega} \frac{\langle N_2 \rangle}{\langle N \rangle} , \quad (17)$$

$$\kappa \cdot \sigma^2 = \kappa_1 \cdot \sigma_1^2 \frac{\omega_1}{\omega} \frac{\langle N_1 \rangle}{\langle N \rangle} + \kappa_2 \cdot \sigma_2^2 \frac{\omega_2}{\omega} \frac{\langle N_2 \rangle}{\langle N \rangle} . \quad (18)$$

Therefore, one can calculate primary fluctuations using Eqs. (6)-(9), (14) and then mix them with the fluctuations of pions from resonance decays using Eqs. (15)-(18).

The limited detector acceptance can be taken into account similar to Refs. [84–86]. In the limit of a very small acceptance window one can neglect all correlations, use binomial distribution for the probability  $q$  for a particle to be accepted,  $0 \leq q \leq 1$ ,  $q \rightarrow 0$ , and obtain

$$\omega = 1 + q(\omega_{\text{all}} - 1) , \quad S \cdot \sigma \simeq 1 + 2q(\omega_{\text{all}} - 1) , \quad \kappa \cdot \sigma^2 \simeq 1 + 6q(\omega_{\text{all}} - 1) , \quad (19)$$

where  $\omega_{\text{all}}$  is the scaled variance for the case when all particles are accepted. One can see from Eq. (19) that for  $\omega_{\text{all}} > 1$  the fluctuations of the accepted particles are always larger than unity and approach to it from above in the small acceptance limit<sup>7</sup>. Equation (19) is an approximation that should be valid for pions from resonance decays, but it is not valid if there is some dependance on  $p_T$ . For example, the relative amount of primary pions from the condensate at  $p = 0$  increases after the application of the cut with  $p_T < 200$  MeV, because they are situated only there [37]. However, Eq. (19) is still useful, because it shows that the increase of acceptance leads, first of all, to the change of higher order fluctuations.

The THERMINATOR model [83] is used for the account of resonances in this paper. Primary particles are sampled with Poisson distribution there, i.e.  $\omega_{\text{prim}} = S_{\text{prim}} \cdot \sigma_{\text{prim}} = \kappa_{\text{prim}} \cdot \sigma_{\text{prim}}^2 \equiv 1$ . It is not correct for primary pions, because their number should be sampled according to Bose-Einstein distribution following Eqs. (6)-(9). However, it gives a good estimate of pion fluctuations due to resonance decays, because resonances are heavy, and one can use Boltzmann statistic for them, see the discussion after Eq. (10). Resonance decays can only increase fluctuations in this case. The effects of resonance decays are stronger for higher temperatures, because of the exponential suppression of heavy particles in TM. The temperature is the highest at  $c = 80-90\%$  centrality, see Fig. (1). Therefore, resonance decays at this centrality give the upper bound on the fluctuations from resonances at all centralities

$$\omega_{\text{res}} \lesssim 1.05, \quad S_{\text{res}} \cdot \sigma_{\text{res}} \lesssim 1.1, \quad \kappa_{\text{res}} \cdot \sigma_{\text{res}}^2 \lesssim 1.3 . \quad (20)$$

Looking at the numerical values in Eq. (20) one can conclude that the scaling (19) holds even quantitatively. Any  $p_T$  cut further decreases the fluctuations for resonances. Therefore, the approximation  $\omega_{\text{res}} = S_{\text{res}} \cdot \sigma_{\text{res}} = \kappa_{\text{res}} \cdot \sigma_{\text{res}}^2 = 1$  is used from here on.

Statistical errors increase extremely fast for normalized skewness and kurtosis when mean multiplicity increases. Using the definitions for the absolute and relative errors of the unknown variable  $X$

$$X = \langle X \rangle \pm \sigma(X), \quad \varepsilon_X = \frac{X - \langle X \rangle}{\langle X \rangle} , \quad (21)$$

one obtains for the mean multiplicity, scaled variance, normalized skewness and kurtosis [91]:

$$\varepsilon_{\langle N \rangle} \simeq \frac{1}{\sqrt{N_{\text{ev}}} \sqrt{\langle N \rangle}}, \quad \varepsilon_{\omega} \simeq \sqrt{\frac{3}{N_{\text{ev}}}}, \quad \varepsilon_{S \cdot \sigma} \simeq \sqrt{\frac{6}{N_{\text{ev}}}} \sqrt{\langle N \rangle}, \quad \varepsilon_{\kappa \cdot \sigma^2} \simeq \sqrt{\frac{24}{N_{\text{ev}}}} \langle N \rangle , \quad (22)$$

where  $N_{\text{ev}}$  is the number of generated events and  $\langle N \rangle$  is the mean multiplicity. Therefore, in order to have a relative error for the normalized kurtosis on the level of  $\varepsilon_{\kappa \cdot \sigma^2} = 10\%$ , one has to generate  $N_{\text{ev}} = 24 * 10^2 * \langle N \rangle^2$  events. For pions in the most central collisions at the LHC it gives the number  $N_{\text{ev}} \sim 10^9$ . For smaller statistics

<sup>7</sup> Global conservation of charges, energy and momentum significantly suppress fluctuations making  $\omega_{\text{all}} < 1$ , see [86–89]. Therefore, in the case when global conservations start to play a role the fluctuations approach to unity from below [90].

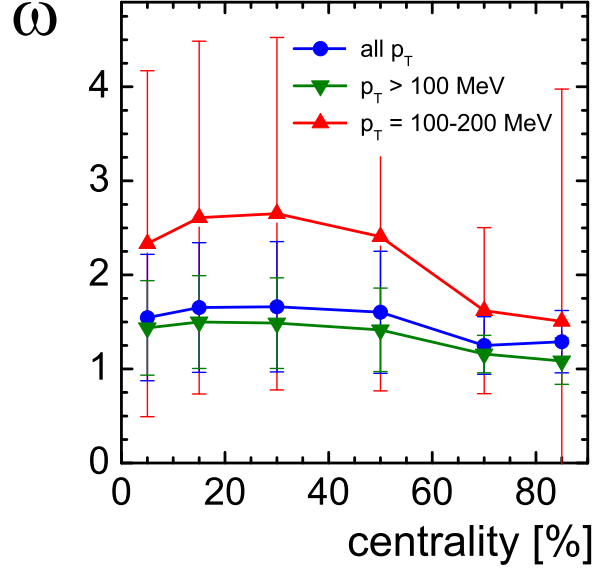


FIG. 2: The total fluctuations of positively (negatively) charged pions. The resonance decays and the condensate for different  $p_T$  cuts as the function of the collision centrality are included.

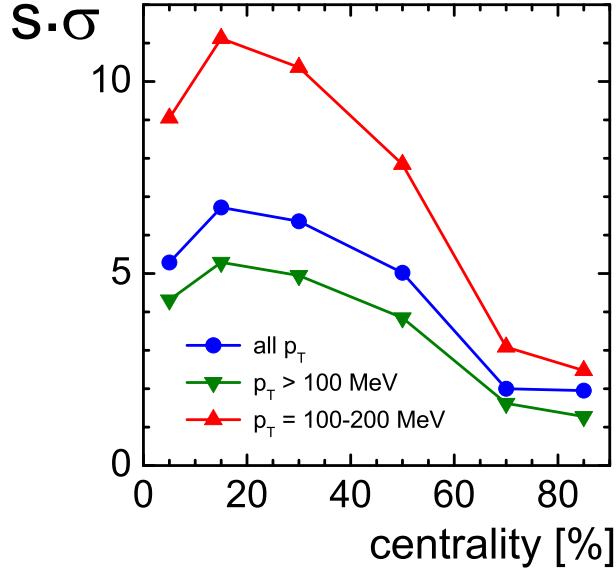


FIG. 3: The same as Fig. 2 for the normalized skewness.

one can obtain huge and even negative values for  $S \cdot \sigma$  and  $\kappa \cdot \sigma^2$  which fluctuate with  $N_{ev}$  just because of small statistics.

The results of the calculations using Eq. (14) are substituted to Eqs. (6)-(9), then to Eqs. (15)-(18), and are presented in Figs. 2-4. The error bars reflect the errors in the  $T$  and  $\mu$  determination from the available experimental data, see Fig. 1 (a), and are shown only for  $\omega$ , see discussion below. The scaled variances increase to some mild values, while the normalized skewness and kurtosis are more sensitive variables. The  $p_T$  cut to  $\Delta p_T = 100 - 200$  MeV gives a factor of 2 increase for the  $S \cdot \sigma$  compared to other cases. The normalized kurtosis reaches the values  $\sim 100$  even for the measured  $p_T$  range, while the  $\Delta p_T = 100 - 200$  MeV further increases it three times to  $\sim 300$ . The scaling between the fluctuations according to Eq. (13) holds for the  $p_T$  cut  $\Delta p_T = 100 - 200$  MeV.

The error bars are about 30%, 40% and 70% of the scaled variance for the ' $p_T > 100$  MeV', the ' $\text{all } p_T$ ', and for the ' $\Delta p_T = 100 - 200$  MeV' cases, correspondingly. They increase, because of the increase of the unknown condensate part in the corresponding cases. The error bars for the  $S \cdot \sigma$  are of the order of 100% and even larger



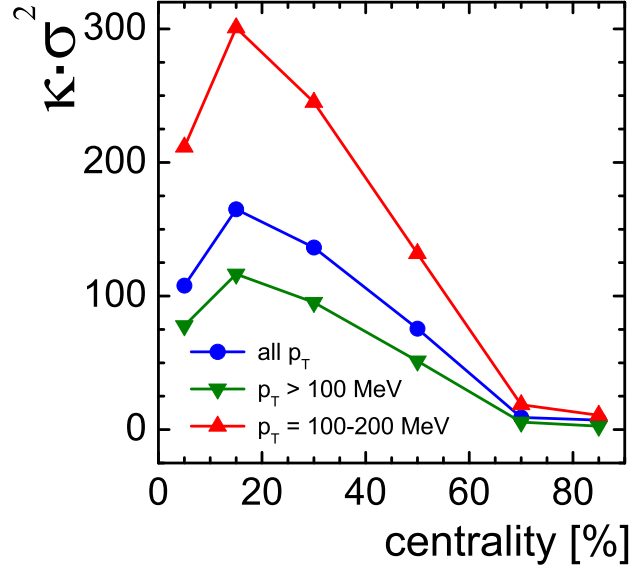


FIG. 4: The same as Fig. 3 for the normalized kurtosis.

for the  $\kappa \cdot \sigma^2$ . So large error bars mean that one can not predict an accurate value of the fluctuations from the current data on mean multiplicities. The experimental measurements of fluctuation may show whether there is pion condensate or not.

Participant number (volume) fluctuation inside of a given centrality is one of the most challenging ingredients of the background. It is large for the scaled variance [92, 93], and strongly increases with the order of fluctuations measure [94]. Therefore, before making any conclusion out of the high order fluctuations data, one should prove that participant number fluctuations are under control.

The effect of cutting the  $p_T$  range to  $\Delta p_T = 100 - 200$  MeV gives much larger effect than measuring pions with all  $p_T$ . It is an important advantage, because decreasing the  $p_T$  requires lower magnetic field and re-calibration of the detectors [49], while a  $p_T$  cut can be implied in the currently used software for the analysis of the events.

## V. CONCLUSIONS

The normalized kurtosis is the most sensitive to chemical non-equilibrium, pion condensation, and any other considered effect. It requires the largest number of measured events, and the knowledge of the tails of the multiplicity distribution. However, it rapidly grows if detector acceptance, size of the system, or relative amount of particles in the condensate increases. It may allow to distinguish between equilibrium and non-equilibrium models at the LHC.

The cut of the transverse momentum  $p_T = 100 - 200$  MeV for positively (negatively) charged pions allows to increase the relative amount of the condensate in the considered events, using already measured data. The possible increase of the normalized kurtosis is so large, that one can check the intriguing possibility of high temperature Bose-Einstein condensation of pions at the LHC experimentally.

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